Characterizing Johnson Noise from Resistors to Approximate Boltzmann Constant

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We have characterized the noise measured from resistors in a circuit known as Johnson noise. At Standard temperature and pressure lower resistor values around 10Ω range failed to line up well with the measured values calculated by Johnson and Nyquist. The noise measured from high value resistors in the 100KΩ range agreed excellently with the predictions of Johnson and Nyquist regarding their work on the thermal agitation of electrons in resistors to determine Boltzmann's constant.

In 2018 John Johnson and Harry Nyquist explained their findings of what they called Johnson-Nyquist Noise. Working at Bell Laboratories, Johnson was able to discover a residual noise in his circuits which was caused by the random spontaneous movement of electricity through a conductor. The electric charges inside a conductor are found to be in a state of thermal agitation, in thermodynamic equilibrium with the heat motion of atoms in the conductor [1]. The manifestation of this phenomenon is a fluctuation of potential difference between the terminals of the conductor which can be measured with suitable instruments [1, 2]. In order to measure this noise one needs to use a suitable amplifier as it it is not readily detectable with standard laboratory instruments.

That being said this noise is often not negligible. A couple of applications affected by Johnson noise include making thermometers that are free of drift(fluctuations in measurement caused by changes in voltage which are usually unexpected) as well as radio frequency setups; which are larger circuits with many components each having their own resistance where the noise can affect the sensitivity of the radios [4]. A basic understanding of the causes and effects of Johnson noise in circuits can give experimentalists a better idea on how their circuits will perform.

The derivation of where Johnson noise comes from is provided by Nyquist in his theorem:

$$V(t)^2 = 4RTk_B \text{ [2]}$$

where the $V(t)^2$ term is known as the mean squared voltage, k_B is known as Boltzmann's constant $= 1.38064852 * 10^{-23} m^2 s^{-2} K^{-1} kg$, R is the value in ohms of the Resistor or in the circuit, and T is the temperature in Kelvin of the Resistor. This equation comes from the analysis of how the power and voltage are affected by the fundamental electron interactions in the resistor [2]. The kinetic energy of electrons and how it is related to the characteristic energy for a given temperature which shown by $\frac{1}{2}mv^2 \approx k_B T$. From this, a velocity v can be obtained which is the velocity of a single electron inside this resistor when using m as the mass of an electron. Because there are many electrons inside any specific resistor a clever usage of the central limit theorem can be invoked to show that the sum of all of the velocities from every electron inside the resistor results in a random net motion of electrons which can be interpreted as current I [4] which in term sets up a voltage V. In their papers, both Johnson and Nyquist go into more detail for this derivation, the information above is all that is needed to understand the results. Because Johnson noise is a random white noise it is not possible to predict the effects of it which makes it difficult to minimize with conventional methods. In order to minimize Johnson noise, experimenters have either the choice to decrease the temperature of the resistor or filter out by reducing the band-with coming into the system. They can not simply reduce the value of the resistor since the power is not a function of the resistor, only the voltage squared $V(t)^2$ is.

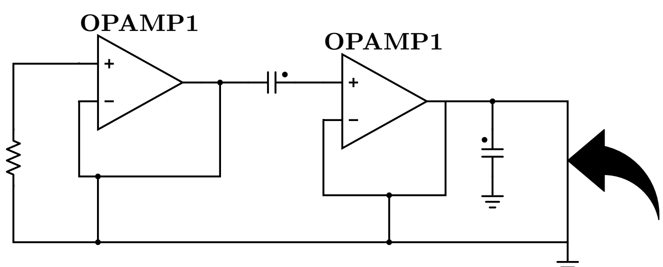

FIG. 1. Shown is the general circuit that was built for testing Johnson noise. If one was to measure the voltage at the point indicated by the arrow, there would in fact be a voltage measured. The resistor on the left hand side is the resistor where the noise would be originating from. The fact that there is no voltage source driving this circuit and there is still a voltage measured is the truly remarkable part of Johnson noise.

Standard lab oscilloscopes lack the fine measurement accuracy needed to interpret Johnson noise. Because of this we used a USB stereo audio adapter that had a 32-bit sample rate with one end connected to laptops and the other via 3.5mm auxiliary cable to BNC straight into our circuit. The sound card included a dial on the front side of it which limited the amount of input coming in. This dial was turned to the maximum setting to allow as much noise as possible in from the circuit. The laptops were running an audio capture program called Audacity (the correct settings for Audacity are crucial, consistent settings throughout measurements is imperative) . With this combination of tools we were able to fully observe, with fine detail, voltage in the circuit. A crucial detail to using this setup was that although the incoming signals were indeed voltage, Audacity is unable to plot voltage vs time graphs. This is where a short python script was written to correctly interpret the signals coming in from the sound card as voltage vs time graphs. We found that our calibration often changed if we returned to the experiment after a period of time, therefore calibrations were conducted before each measurement.

When connecting the resistor to the circuit, sufficient shielding is needed to keep out additional noise from affecting the noise coming from the resistor. An example of this type of stray noise is the "wall frequency" which has peaks at every 60hz interval. A resistor was connected to the receiving end of a male BNC connection. The resistor was then surrounded by a 1/16" copper shielding. This provided the necessary shielding for Johnson noise to be the only noise we measured

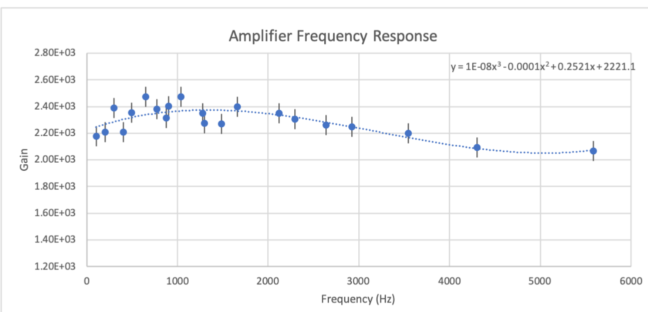

FIG. 2. Shown is the frequency response calculations for the amplifier. As seen from the results there is a narrow range where the amplifier gives a consistent range. This is band from 550Hz to 1700Hz was chosen.

Amplifying the noise coming from the resistor was implemented by using a high gain pre-amplifier. A pre-amp was used because of its ability to boost low voltages to high voltages. The gain of the amplifier used was approximately 2370 times the input. This was measured over a broad frequency range to visualize areas of consistent gain. The amplifier used in this specific circuit provided a frequency response that was not as smooth as we would have liked it to be over the expected range as shown in figure 2. Because of this we decided to use a relatively small frequency range of about 1100Hz for our measurements.

The limiting of the frequency range affects how the power spectral density (PSD) graph is interpreted as the voltage squared is determined by taking an average of the V^2/Hz values over a specific range of frequencies. The PSD graph has the y-axis of V^2/Hz and x-axis of frequency in Hz. This graph is created by taking the noise coming in from the sound card and audacity setup in as voltage V vs time (s) graph and applying a Fourier transform which breaks down the noisy signal in such a way that it can be interpreted as a V^2/Hz vs Hz graph. This Fourier transformed graph has a large variety of information that can be extrapolated from it, the relevant information for this experiment are the V^2/Hz values in the specified frequency range. The average of these values gives us the calculated $V^2(t)$ value that we can now use in the equation: $V(t)^2 = 4RTk_B$, solving for k_B and seeing how close to the expected values we would get.

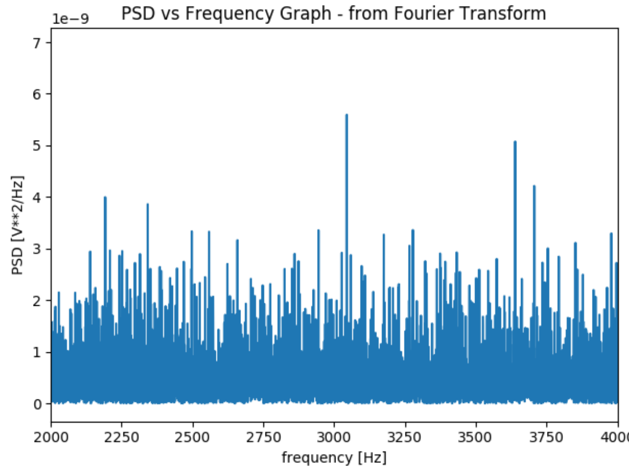

FIG. 3. Shown is the noise coming coming from the resistor in terms of voltage V vs time transformed to the PSD V^2/Hz vs frequency Hz. The range of the x-axis is 100 Hz to 500Hz which is the range that we used to calculate the average PSD value.

The Data collected of the Johnson noise measured from various resistors is shown in the table below. The exact value of Boltzmann's constant is $1.38064852 * 10^{-23} m^2 s^{-2} K^{-1} kg$. Error calculations were carried out throughout the experiment to get the final error for the final calculated k_B values. The reasons for discrepancies are likely due to failure of correct calibration of the equipment used. As seen the data collected from the resistors greatly affected how close to the expected value of Boltzmann's constant was obtained. To clean the data and we removed values of resistors that were not close to what we expected.

A possible reason for these resistors giving use skewed values could be the fact that the noise from the amplifier part of the circuit is dominating, and therefore affecting R value in the $V^2(t)$ equation. As seen the average of the cleaned resistor values subtracted from actual Boltzmann's constant gives a difference of $6.23E - 26 k_B units$. This value shows how much the calculated value differs from the expected value. When including the k_B values from resistors of low values (in the range of 12Ω to 1000Ω the difference between the expected values is greater than $\pm 10E - 23 k_B units$. The values measured here indicate that Johnson noise can indeed be measured with accuracy and can be used to determine Boltzmann's constant. Work done by both Johnson and Nyquist show promising results and can be experimentally reproduced.

Resistance (ohms)	V2/deltaf	Calculated k_B
201.8	2.50E-17	1.05-22 ± 1.3E-22
1790	3.66E-17	1.74E-23 ± 1.3E-23
3290	5.74E-17	1.48E-23 ± 1.3E-23
6788	1.11E-16	1.39E-23 ± 1.3E-23
14870	2.39E-16	1.36E-23 ± 1.3E-23
30000	4.53E-16	1.28E-23 ± 1.3E-23
108000	1.61E-15	1.79E-23 ± 1.3E-23
179000	2.72E-15	1.29E-23 ± 1.3E-23
270000	4.58E-15	2.01E-23 ± 1.3E-22
301000	4.36E-15	1.23E-23 ± 1.3E-23
390000	5.79E-15	1.26E-23 ± 1.3E-23

FIG. 4. Shown are the resistor values, average PSD values, and calculated Boltzmann constant k_B values each with their respective units.

The circuit used for the measurement portion of the experiment is a simple circuit with not many components. That being said, each component has parts of it that also contribute to the noise detected. A way to improve upon the results would be to filter out the noise that is coming from the amplifier, reducing the length of the wires and thorough calibration of the equipment used. Filtering out the noise from the amplifier circuit could be be done by incorporating a dual channel set up for the amplifier, allowing the amplifier noise to be quantified and removed from the measurement. Shorter wires would allow for less noise since longer wires can act as "antenna". Proper calibration would entail conducting the sound card calibration each time a measurement was being taken along with recalculating the gain provided by the amplifier. Knowing these values precisely would likely provide values that are much closer to the expected.

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